When to Go Negative in Political Campaigns? *

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Abstract

We consider a political competition in which a privately informed incumbent has the option to go negative—disclose a scandal—in their campaign against a challenger. The voter has uncertainty about two dimensions of attributes: the ability and corruption of the challenger, and will receive an exogenous signal about the ability before voting. We focus on the separating equilibrium in which the disadvantaged type goes negative and competes on the corruption dimension, while the advantaged type prefers to compete on the ability dimension. We show that the voter may be worse off by having a more precise signal about the challenger's ability, as this may lead to less information being revealed on the dimension controlled by the incumbent.

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1 Introduction

Negative campaigning is not a new feature of elections, but has recently become more prominent. The term "negative" generally refers to a type of political campaign that "attacks the other candidate personally, the issues for which the other candidate stands, or the party of the other candidate" (Surlin and Gordon, 1997). Politicians invest significant resources in uncovering and spreading negative information about their opponents to attract more votes. However, given that much of this information is unverifiable, it is difficult to assess beforehand the impact a scandal will have on the voter's decision by the time the election arrives. Often, politicians who contemplate using negative campaigning are unsure about its ultimate impact on voters. In some cases, the victim of a negative campaign may even become more popular afterward.

In this paper, we study a model in which a voter chooses between an incumbent politician and a challenger. There are two dimensions of uncertainty about the challenger, which are ability and corruption, both valued by the voter. We assume that throughout the election campaign, the voter learns about the challenger's attributes, while the voter's utility from the incumbent remains fixed.

We explore whether and when it is optimal for an incumbent politician, who is privately informed about the challenger's ability, to engage in negative campaigning by disclosing a signal (referred to as a "scandal") regarding the challenger's corruption. When deciding on a campaign strategy, the incumbent takes into account that the voter will also receive a signal about the challenger's ability before voting. There is ex-ante uncertainty regarding the voter's reaction to the scandal; if it turns out to be effective (not effective), then the voter's posterior about the challenger being corrupt increases (decreases).

Our analysis focuses on the existence of a separating equilibrium in which the decision to disclose a signal depends on the incumbent's private information about the ability of the challenger, about which the voter also gets a signal before the election.

There is a unique separating equilibrium that exists under certain conditions: when the incumbent's private signal is sufficiently informative and the corruption parameter is important enough. If disclosed, the corruption parameter must be decisive in the voter's decision, i.e., overcome the effect of the public information on ability.

In this separating equilibrium, it is optimal for the incumbent not to disclose the signal about the scandal if and only if the challenger does not pose a significant threat to the incumbent. This is the case when the incumbent has a low signal about the challenger's ability. Anticipating that the voter is also likely to get a low signal about the challenger during the election campaign, the incumbent does not want to disclose a scandal. On the other hand, the incumbent, who realizes that the voter is likely to get a high ability signal about the challenger, prefers to disclose the signal. We show that this is the only possible separating equilibrium (Proposition 1), which exists if and only if the signal is informative enough and the corruption parameter is important enough in the voter's utility. The separating equilibrium is more likely to arise when the incumbent's private information and the voter's public signal about the ability that will arrive before the election are more informative. Also, the region for this equilibrium is highest when the uncertainty about the ability is highest, while it decreases as the belief about ability goes to 0 or 1, hence as uncertainty about ability resolves (Proposition 2).

When the effectiveness of the corruption signal is insufficient, even the disadvantaged type does not find it worthwhile to disclose a scandal. In this case, neither type of incumbent discloses the exogenous signal and competes purely on the ability parameter. Consequently, the voter only learns about the challenger's ability and only via the exogenous public signal that is revealed through the campaign.

When the effectiveness of the corruption signal is very high, i.e., it is very likely that a scandal turns out to be effective, then both types of the incumbent disclose the exogenous signal. In this case, the voter learns about both the corruption scandal and the public signal about ability.

We also present welfare comparisons for the incumbent and the voter. As the precision of the incumbent's private signal (or the precision of the voter's signal) increases, the separating equilibrium is more likely to arise as the advantaged type prefers to compete on the ability dimension while the disadvantaged type prefers to compete on the corruption dimension. We find that the voter's welfare comparison between separating equilibrium and pooling on revealing is not straightforward; it depends on the relative importance of the two attributes in the voter's utility. Pooling on revealing dominates separating equilibrium if and only if the corruption parameter is sufficiently important relative to ability (Proposition 3). The trade-off is that, in separating equilibrium, the voter will learn about both the incumbent's private information and sometimes about the corruption parameter. In contrast, in the pooling equilibrium on revealing, the voter always learns about the corruption parameter but never about the incumbent's private information. The public information about ability is present in any equilibrium. Based on this result, we conclude that the voter's welfare may decrease as the precision of their information, denoted by π_3, increases, leading to a shift from pooling on revealing to a separating equilibrium.

Contrary to the common belief about negative campaigning, our model demonstrates that negative campaigns can lead to better-informed decision-making by the voter in some cases.

Our main application is in political elections, but our model applies to other settings as well. It can be viewed as a signaling model with two dimensions and outside information, where only one dimension can be controlled by a privately informed sender, and the receiver has access to an exogenous signal about the other dimension. Our results show that the receiver can be worse off by having access to more precise outside information.

1.1 Related Literature

Surprisingly, negative campaigning is not as extensively studied in the political economy literature as one might expect. Most previous literature focuses on the trade-off between allocating resources to positive versus negative campaigning. Our innovation is to introduce a model in which a privately informed incumbent has the option to disclose a signal on a different dimension. By offering this choice, we allow the incumbent to choose the issue that dominates the voter's decision during the election campaign.

Skaperdas and Grofman (1995) find that the front-runner engages in less negative and more positive campaigning. In their setup, politicians are endowed with a unit of resources to divide between positive and negative campaigning. Positive campaigning aims to attract voters, while negative campaigning seeks to reduce the opponent's votes. In equilibrium, the front-runner tends to engage less in negative campaigning. Similarly, Harrington and Hess (1996) consider a model where politicians allocate resources between positive and negative campaigning, finding that the candidate with less attractive attributes engages more in negative campaigning. The two dimensions in their model are ideology and personal, where only ideology can be influenced via costly relocation. Again, the impact of the campaign is direct, which is through moving the location of the party's ideology away from the median voter. Polborn and Yi (2006) consider a model where politicians choose between positive and negative campaigning with no uncertainty about the effect of the negative campaign. Each politician chooses whether to reveal their own (positive) or their opponent's (negative) valence dimension. Information is truthful, and there is no uncertainty about the component that is revealed.

A common theme in previous work is the direct impact of negative campaigning on voters' choice, while in this paper, the effect is through indirect signaling. The use of a scandal is a high-risk, high-reward strategy! In our setup, there is no trade-off between positive and negative campaigning. Another main departure is that the incumbent's decision to reveal a scandal depends on his private information. Negative campaigning leads to voter learning on both dimensions of uncertainty about the challenger.

Nakaguma and Souza (2022) empirically show that candidates who are leading in the polls are more likely to be the target of electoral campaign attacks. Dziuda and Howell (2021) and Ogden and Medina (2020) also consider the use of political scandals in political competition, in setups different from ours.

2 The Model

Consider three players: an incumbent (I), a challenger (C), and a representative voter (V). The voter obtains a fixed utility from voting for the incumbent, and there is no further learning about this politician. In contrast, the voter faces two dimensions of uncertainty about the challenger: ability $\theta \in \{\theta_L, \theta_H\}$ and corruption $\omega \in \{0, 1\}$. The challenger has high ability ($\theta = \theta_H$) with probability p_θ, and is corrupt ($\omega = 1$) with probability p_ω. All signals are about the challenger, so the voter only updates his beliefs about θ and ω.

The incumbent privately observes a signal, s_1, about the challenger's ability, and the voter will also observe a signal, s_3, about the same parameter **before** voting. The signal s_2 provides information about corruption and is available to the incumbent at zero cost. The realization of s_2 is observed by the voter only if the incumbent chooses to reveal it. When deciding whether or not to reveal s_2, the incumbent does not know the realization of this signal or that of signal s_3, and shares the same belief as the voter regarding the challenger's corruption. The incumbent is of two types depending on the realization of s_1: the incumbent is referred to as the *disadvantaged type* if $s_1 = H$, and the *advantaged type* if $s_1 = L$.

Signals

(i) The signal $s_1 \in \{L, H\}$ about the challenger's ability θ has precision π_1, where $\Pr(s_1 = L \mid \theta_L) = \Pr(s_1 = H \mid \theta_H) = \pi_1$.

(ii) The signal $s_2 \in \{weak, strong\}$ about the challenger's corruption ω is revealed only if the incumbent chooses to do so, where $\Pr(s_2 = weak \mid \omega = 0) = \Pr(s_2 = strong \mid \omega = 1) = \pi_2$. If the incumbent does not reveal the signal, then $s_2 = \emptyset$.

(iii) The signal $s_3 \in \{l, h\}$ about θ has precision π_3, where $\Pr(s_1 = l \mid \theta_L) = \Pr(s_1 = h \mid \theta_H) = \pi_3$.

Assumptions

The ability θ and corruption ω are independent. The signals s_1 and s_3 depend only on θ and are conditionally independent given θ. The signal s_2 depends only on ω and is independent of s_1 and s_3. We assume that $\pi_1, \pi_2, \pi_3 \in (0.5, 1)$.

Utilities

Both the incumbent and challenger receive a utility of 1 if elected, and 0 otherwise. They care only about winning with no associated costs. The voter receives a constant utility from voting for the incumbent, u_0, which is commonly known at the beginning of the game. The voter's utility from voting for the challenger is linear in ability θ and corruption ω, denoted by $u(\theta, \omega) = k \Pr(\theta = \theta_H) - \Pr(\omega = 1)$, where $k > 0$ represents the relative importance of ability compared to corruption.

Strategies

(i) The incumbent decides whether or not to reveal the scandal: $s_1 \to R \in \{0, 1\}$.

(ii) The voter chooses whether to vote for the incumbent or the challenger: $R \times s_2 \times s_3 \to v \in \{I, C\}$.

Timing of the Game

At $t = 0$, nature determines the challenger's ability, $\theta \in \{\theta_L, \theta_H\}$, and corruption status, $\omega \in \{0, 1\}$.

At $t = 1$, the incumbent privately observes a noisy signal $s_1 \in \{L, H\}$ about θ. Then, the incumbent decides whether or not to reveal s_2: $R \in \{0, 1\}$.

At $t = 2$, if the incumbent has revealed the scandal ($R = 1$), the voter observes a signal $s_2 \in \{weak, strong\}$ about ω. If the incumbent did not reveal the scandal

($R = 0$), no information about the challenger's corruption is available to the voter. Additionally, the voter receives a noisy signal $s_3 \in \{l, h\}$ about θ.

At $t = 3$, the voter makes a voting decision $v \in \{I, C\}$.

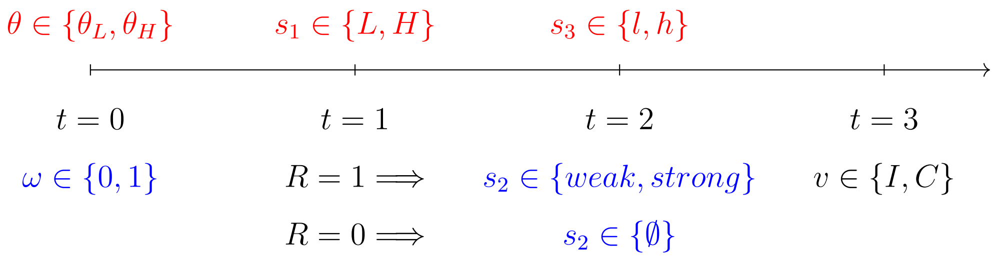

Figure 1: Timing of the game.

Discussion on the Signal s_2

When the incumbent decides whether or not to reveal signal s_2, he is uncertain about its realization but knows its precision π_2. We refer to signal s_2 as a *scandal*, with its realization representing the impact of the scandal on the voter's decision. Several factors may influence the effectiveness of a reported scandal, such as media coverage, investigation by journalists, subsequent actions by the challenger, and the voters' attention and response to the news. Negative campaigning, which involves revealing scandals, is inherently risky and can backfire if the allegations are perceived as weak or unfounded.

If $s_2 = strong$, it implies that the reported scandal is seen as credible and persuasive, increasing the voter's belief that the challenger is corrupt. On the other hand, if $s_2 = weak$, it indicates that the scandal lacks credibility, reducing the voter's belief in the challenger's corruption. The incumbent has a commonly known expectation about the likelihood of the scandal being effective. A higher precision π_2 signifies greater media accuracy, more thorough scrutiny by the public, and higher political awareness among voters. Thus, when π_2 is high, the likelihood that the scandal will successfully shift voter opinion increases, making the revelation of π_2 a more potent political tool.

3 Equilibrium Analysis

In this model, we focus on pure-strategy perfect Bayesian equilibria. The voter can rely on the incumbent's revealing strategy to infer the realization of signal s_1, which is privately observed by the incumbent. The voter's utility from the challenger, $u(\theta, \omega)$, can be expressed as a function of the signal realizations:

$$u(s_1, s_2, s_3).$$

The voter will choose to vote for the incumbent if $u_0 > u(s_1, s_2, s_3)$, and the challenger otherwise, breaking a tie with probability 0.5.

Assumption 1. *$u(H, l) < u_0 < u(L, h)$.*

This assumption ensures that the voter's utility u_0 from the incumbent is bounded by his utility from the challenger. It guarantees that neither type of the incumbent is automatically elected based solely on the realizations of signals s_1 and s_3.

Below, we summarize the main equilibria. A more detailed analysis of the equilibria is provided in the Appendix.

3.1 Separating Equilibrium

The incumbent is considered disadvantaged if $s_1 = H$, and advantaged if $s_1 = L$.

Proposition 1 (Separating Equilibrium)**.** *The only separating equilibrium occurs when the disadvantaged type reveals the scandal (signal s_2), while the advantaged type does not. If the scandal is revealed, the voter will vote for the incumbent if and only if $s_2 = strong$; otherwise, the voter votes for the incumbent if and only if $s_3 = l$:*

$$\Pr(v = I \mid s_2 = \mathit{strong}) = \Pr(v = I \mid s_3 = l) = 1.$$

This equilibrium exists if

$$Pr(s_3 = l \mid s_1 = H) < Pr(s_2 = strong) < Pr(s_3 = l \mid s_1 = L), \tag{1}$$

and

$$u(H, h, strong) < u_0 < u(H, l, weak). \tag{2}$$

In this separating equilibrium, the voter's decision depends on the realization of s_2 if it is disclosed, and on s_3 otherwise. This implies that the scandal signal s_2 is precise enough to outweigh the public signal s_3 whenever it is revealed. Thus, the incumbent chooses which issue the voter focuses on when making their voting decision via their disclosure strategy.

Condition (2) implies that

$$k < \frac{p_\omega{}^s - p_\omega{}^w}{p_\theta{}^{Hh} - p_\theta{}^{Hl}},$$

indicating that k—the weight of ability relative to corruption—is not too high. In other words, the voter's focus on ability must not overshadow the importance of corruption.

3.2 Pooling Equilibrium on Revealing

There are two types of pooling equilibria on revealing the scandal.

1. The first pooling equilibrium can coexist with the separating equilibrium. In this case, both types of the incumbent reveal the scandal, and the incumbent is elected if and only if $s_2 = strong$ or $s_2 = weak$ and $s_3 = l$:

   $$\Pr(v = I \mid h, strong) = \Pr(v = I \mid l, strong) = \Pr(v = I, l, weak) = 1.$$

   When the incumbent deviates from this strategy, for any off-equilibrium belief, the voter elects the incumbent if and only if $s_3 = l$.

In this equilibrium, both types of the incumbent strictly prefer to disclose the scandal for any values of $\Pr(s_2 = \textit{strong})$. However, for this equilibrium to hold when the separating equilibrium also exists, it must satisfy:

$$u(l, \textit{weak}) < u_0.$$

Given that we assume $u(l, \textit{weak}) > u_0$, we discard this type of equilibrium.

2. The second pooling equilibrium exists outside the separating equilibrium region, for:

$$\Pr(s_2 = \textit{strong}) > \Pr(s_3 = l \mid s_1 = L).$$

In this case, both types of the incumbent reveal the scandal, and the incumbent is elected if and only if $s_2 = \textit{strong}$:

$$\Pr(v = I \mid h, \textit{strong}) = \Pr(v = I \mid l, \textit{strong}) = 1.$$

When the incumbent deviates from this strategy, for any off-equilibrium belief, the voter elects the incumbent if and only if $s_3 = l$.

3.3 Pooling Equilibrium on Not Revealing

When $\Pr(s_2 = \textit{strong}) < \Pr(s_3 = l \mid s_1 = H)$, even the disadvantaged type prefers not to reveal the scandal. In this case, the voter votes for the incumbent if and only if $s_3 = l$. When the incumbent deviates and reveals the scandal, for any off-equilibrium belief, the voter votes for the incumbent when $s_2 = \textit{strong}$ and votes for the challenger when $s_2 = \textit{weak}$.

This pooling equilibrium, where the scandal is not revealed, cannot coexist with the separating equilibrium. Detailed proofs are provided in the Appendix.

4 Comparative Statics

This section focuses on comparative statics.

4.1 Comparative Statics on Threshold Values

We will now conduct a comparative statics analysis of the threshold values necessary for the existence of different equilibria, in relation to the model's parameters. To do so, we introduce two key thresholds, defined as follows:

Let $\underline{\lambda}$ be the conditional probability of signal realization $s_3 = l$ given $s_1 = H$, where:

$$\begin{aligned}\underline{\lambda} &\triangleq \Pr(s_3 = l \mid s_1 = H) \\ &= \frac{p_\theta \pi_1 (1-\pi_3) + (1-p_\theta)(1-\pi_1)\pi_3}{p_\theta \pi_1 + (1-p_\theta)(1-\pi_1)}.\end{aligned}$$

Similarly, let $\overline{\lambda}$ be the conditional probability of signal realization $s_3 = l$ given $s_1 = L$, where:

$$\begin{aligned}\overline{\lambda} &\triangleq \Pr(s_3 = l \mid s_1 = L) \\ &= \frac{p_\theta (1-\pi_1)(1-\pi_3) + (1-p_\theta)\pi_1 \pi_3}{p_\theta (1-\pi_1) + (1-p_\theta)\pi_1}.\end{aligned}$$

The region for the separating equilibrium is illustrated in Figure 2. When the value of $\Pr(s_3 = strong)$ lies between the two thresholds $\underline{\lambda}$ and $\overline{\lambda}$, the separating equilibrium is possible.

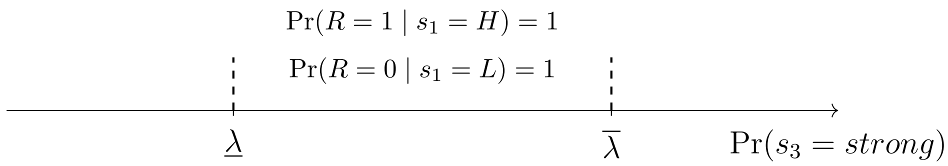

Figure 2: The region for the separating equilibrium.

Define the difference between the two thresholds as $D \triangleq \overline{\lambda} - \underline{\lambda}$, where:

$$D = \frac{p_\theta(p_\theta - 1)(2\pi_1 - 1)(2\pi_3 - 1)}{(p_\theta + \pi_1 - 2p_\theta\pi_1)(1 - p_\theta - \pi_1 + 2p_\theta\pi_1)}.$$

Proposition 2. *The following properties hold for D:*

(i) $D = 0$ *if* $p_\theta = 0$ *or* $p_\theta = 1$.

(ii) $\frac{\partial D}{\partial p_\theta} > 0$ *if* $p_\theta < 0.5$ *and* $\frac{\partial D}{\partial p_\theta} < 0$ *if* $p_\theta > 0.5$.

(iii) $D = (2\pi_1 - 1)(2\pi_3 - 1)$ *if* $p_\theta = 0.5$, *which is the maximum value of* D.

(iv) $\frac{\partial D}{\partial \pi_1} > 0$,

(v) $\frac{\partial D}{\partial \pi_3} > 0$.

The difference between the two thresholds is the widest when $p_\theta = 0.5$ and decreases as p_θ approaches 0 or 1, where it ultimately shrinks to 0. With the most uncertainty about the challenger's ability, the region for separating equilibrium is maximal, and the informational value of the incumbent's private signal (s_1) about the challenger's ability is highest. When there is no uncertainty about the challenger's ability, the separating equilibrium does not exist, and the informational value of s_1 is zero.

As the precision of the signals regarding the challenger's ability (π_1 or π_3) increases, the region for the separating equilibrium widens. The intuition is as follows: When the incumbent's private signal (s_1) becomes more accurate (π_1 increases), or the voter's public signal (s_3) improves in accuracy (π_3 increases), the disadvantaged type ($s_1 = H$) anticipates that the voter is more likely to receive a high-ability signal ($s_3 = h$) about the challenger. As a result, the incumbent is more likely to reveal the scandal and compete on the dimension of corruption. On the other hand, the advantaged type ($s_1 = L$) infers that the voter is more likely to receive a low-ability signal ($s_3 = L$) about the challenger, and thus avoids revealing the scandal, preferring to compete on the dimension of ability.

4.2 Comparative Statics on Payoffs

We now conduct comparative statics on the incumbent's and the voter's payoffs.

Incumbent's payoff

In the separating equilibrium, the incumbent's ex-ante payoff is given by:

$$\begin{aligned}&\Pr(s_1 = H)\Pr(s_2 = strong) + \Pr(s_1 = L)\Pr(l \mid L)\\ =&(p_\omega\pi_2 + (1-p_\omega)(1-\pi_2))(p_\theta\pi_1 + (1-p_\theta)(1-\pi_1))+\\ &(1-p_\theta)\pi_1\pi_3 + p_\theta(1-\pi_1)(1-\pi_3).\end{aligned}$$

The derivative of this payoff with respect to π_1:

$$\Pr(s_2 = strong)(2p_\theta - 1) + \pi_3 - p_\theta > 0,$$

is positive when $p_\theta = 0.5$.

The derivative with respect to π_2 is positive if $p_\omega > 0.5$ and negative otherwise.

The derivative with respect to π_3 is given by:

$$\pi_1 - p_\theta,$$

which will be positive if $\pi_1 > p_\theta$.

In the pooling equilibrium on revealing, the incumbent wins only when $s_2 = strong$ and his utility is given by

$$\Pr(s_2 = strong) = p_\omega\pi_3 + (1-p_\omega)(1-\pi_3) = 1 - p_\omega - \pi_3 + 2p_\omega\pi_3.$$

This payoff increases in π_2 if $p_\omega > 0.5$ and decreases in π_2 otherwise.

In the pooling equilibrium of not revealing, the incumbent wins only if $s_3 = l$ and his utility is given by

$$\Pr(s_3 = l) = p_\theta \pi_3 + (1 - p_\theta)(1 - \pi_3).$$

This payoff increases in π_3 if $p_\theta > 0.5$ and decreases in π_3 otherwise.

Voter's Payoff

In the separating equilibrium, the voter's payoff is given by:

$$\Pr(s_1 = L)(\Pr(s_3 = l \mid s_1 = L)u_0 + \Pr(s_3 = h \mid s_1 = L)(k{p_\theta}^{Lh} - p_\omega))+ \\ \Pr(s_1 = H)(\Pr(s_2 = strong)u_0 + \Pr(s_2 = weak)(k{p_\theta}^{H} - {p_\omega}^{w})).$$

This can be expanded as:

$$((1-\pi_1)(1-\pi_3)p_\theta+(1-p_\theta)\pi_1\pi_3)u_0+(p_\theta(1-\pi_1)\pi_3+(1-p_\theta)\pi_1(1-\pi_3))(k{p_\theta}^{Lh}-p_\omega)+ \\ (p_\theta\pi_1 + (1 - p_\theta)(1 - \pi_1))(\Pr(s_2 = strong)u_0 + \Pr(s_2 = weak)(k{p_\theta}^{H} - {p_\omega}^{w})).$$

The derivative with respect to π_1 is given by:

$$\Pr(s_1 = L)(-k\pi_3 + (-1 + 2\pi_3)(p_\omega + u_0)) + \Pr(s_1 = H)k \Pr(s_2 = weak),$$

but it is unclear whether this is positive.

The derivative of this payoff with respect to π_3 is positive, since

$$\Pr(s_1 = L)(k(1 - \pi_1) + (2\pi_1 - 1)(p_\omega + u_0)) > 0.$$

In the pooling equilibrium on revealing, the voter's payoff is given by:

$$\Pr(s_2 = strong)u_0 + \Pr(s_2 = weak)(kp_\theta - {p_\omega}^{w}),$$

which is unaffected by the precision of the signals regarding the challenger's ability.

In the pooling equilibrium of not revealing, the voter's payoff is:

$$(p_\theta \pi_3 + (1-p_\theta)(1-\pi_3))(k{p_\theta}^h - p_\omega) + (p_\theta(1-\pi_3) + (1-p_\theta)\pi_3)u_0.$$

This payoff is not influenced by π_1. The derivative of this payoff with respect to π_3 is:

$$-0.5u_0 + \pi_3 k,$$

which is positive when $\pi_3 \geq 0.5$ and $u_0 < k$.

Proposition 3. *The voter's ex-ante utility in the pooling equilibrium on revealing is higher than that in the separating equilibrium if and only if:*

$$k < \frac{p_\omega - {p_\omega}^w}{{p_\theta^{L}}^h - p_\theta + (p_\theta^H - p_\theta)\frac{\Pr(s_1=H)}{\Pr(s_1=L)}}.$$

If k is low enough—indicating that corruption is sufficiently important compared to ability—having more precise information about ability (higher π_3) may hurt the voter, as the equilibrium transitions from pooling to separating.

5 Conclusion

In this paper, we consider a political competition where a privately informed incumbent has the option to disclose a scandal about a challenger. The voter faces uncertainty regarding two dimensions of the challenger's attributes: ability and corruption, and will receive an exogenous signal about ability before voting. We show that the increased precision of this exogenous signal can paradoxically make the voter worse off. This occurs because it may result in less information being revealed regarding the corruption dimension controlled by the incumbent.

A Appendix

A.1 Belief Updating

The belief updating upon signal realization is as follows:

$p_\theta^H \triangleq \Pr(\theta = \theta_H \mid s_1 = H) = \frac{p_\theta \pi_1}{p_\theta \pi_1 + (1-p_\theta)(1-\pi_1)}$;

$p_\theta^L \triangleq \Pr(\theta = \theta_H \mid s_1 = L) = \frac{p_\theta(1-\pi_1)}{p_\theta(1-\pi_1) + (1-p_\theta)\pi_1}$;

$p_\theta^{Hh} \triangleq \Pr(\theta = \theta_H \mid s_1 = H, s_3 = h) = \frac{p_\theta \pi_1 \pi_3}{(1-p_\theta)(1-\pi_3) + \pi_1(1-\pi_3-p_\theta)}$;

$p_\theta^{Hl} \triangleq \Pr(\theta = \theta_H \mid s_1 = H, s_3 = l) = \frac{p_\theta \pi_1(1-\pi_3)}{p_\theta \pi_1 + \pi_3(1-\pi_1-p_\theta)}$;

$p_\theta^{Lh} \triangleq \Pr(\theta = \theta_H \mid s_1 = L, s_3 = h) = \frac{p_\theta(1-\pi_1)\pi_3}{p_\theta \pi_3 + \pi_1(1-\pi_3-p_\theta)}$;

$p_\theta^{Ll} \triangleq \Pr(\theta = \theta_H \mid s_1 = L, s_3 = l) = \frac{p_\theta(1-\pi_1)(1-\pi_3)}{\pi_1 \pi_3 + p_\theta(1-\pi_1-\pi_3)}$;

$p_\omega^w \triangleq \Pr(\omega = 1 \mid s_2 = weak) = \frac{p_\omega(1-\pi_2)}{p_\omega(1-\pi_2) + (1-p_\omega)\pi_2}$;

$p_\omega^s \triangleq \Pr(\omega = 1 \mid s_2 = strong) = \frac{p_\omega \pi_2}{p_\omega \pi_2 + (1-p_\omega)(1-\pi_2)}$.

If $p_\theta = p_\omega = 0.5$, we have:

$p_\theta^H = \pi_1$;

$p_\theta^L = 1 - \pi_1$;

$p_\theta^{Hh} = \frac{\pi_1 \pi_3}{\pi_1 \pi_3 + (1-\pi_1)(1-\pi_3)}$;

$p_\theta^{Hl} = \frac{\pi_1(1-\pi_3)}{\pi_1(1-\pi_3) + (1-\pi_1)\pi_3}$;

$p_\theta^{Lh} = \frac{(1-\pi_1)\pi_3}{(1-\pi_1)\pi_3 + \pi_1(1-\pi_3)}$;

$p_\theta^{LL} = \frac{(1-\pi_1)(1-\pi_3)}{(1-\pi_1)(1-\pi_3) + \pi_1 \pi_3}$;

$p_\omega^w = 1 - \pi_2$;

$p_\omega^s = \pi_2$.

A.2 Equilibrium Analysis

We focus on pure strategy equilibria in which the probability of being elected is either 0 or 1.

A.2.1 Separating Equilibria

First, consider the separating equilibrium in which the disadvantaged type always reports the scandal, $\Pr(R = 1 \mid s_1 = H) = 1$, while the advantaged type never reports, $\Pr(R = 0 \mid s_1 = L) = 1$.

In separating equilibrium, the incumbent's reporting strategy $R \in \{0, 1\}$ reveals his private signal $s_1 \in \{H, L\}$.

Let $\Pr(v = I \mid H, s_2, s_3)$ be the probability that the disadvantaged type is elected, after the voter observes the realization $s_2 \in \{strong, weak\}$ of reported scandal $(R = 1)$ and the signal $s_3 \in \{h, l\}$ about the challenger's ability.

$\Pr(v = I \mid H, s_2, s_3) = 1(0)$ if and only if the voter's utility, u_0, from the incumbent is higher (lower) than the utility, $u(H, s_2, s_3)$, from the challenger.

Let $\Pr(v = I \mid L, s_3)$ be the probability that the advantaged type is elected, after the voter observes the signal $s_3 \in \{h, l\}$ about the challenger's ability, when no scandal is reported $R = 0$.

$\Pr(v = I \mid L, s_3) = 1(0)$ if and only if the voter's utility, u_0, from the incumbent is higher (lower) than the utility, $u(L, s_3)$, from the challenger.

The advantaged type $(s_1 = L)$ prefers not to report the scandal if the utility from not reporting is weakly higher than the utility from reporting:

$$\Pr(v = I \mid L, h) \Pr(s_3 = h \mid s_1 = L) + \Pr(v = I \mid L, l) \Pr(s_3 = l \mid s_1 = L)$$
$$\geq \Pr(v = I \mid H, h, \mathit{strong}) \Pr(s_3 = h \mid s_1 = L) \Pr(s_2 = \mathit{strong})$$
$$+ \Pr(v = I \mid H, l, \mathit{strong}) \Pr(s_3 = l \mid s_1 = L) \Pr(s_2 = \mathit{strong})$$
$$+ \Pr(v = I \mid H, h, \mathit{weak}) \Pr(s_3 = h \mid s_1 = L) \Pr(s_2 = \mathit{weak})$$
$$+ \Pr(v = I \mid H, l, \mathit{weak}) \Pr(s_3 = l \mid s_1 = L) \Pr(s_2 = \mathit{weak})$$

The disadvantaged type ($s_1 = H$) prefers to report the scandal if the utility from reporting is weakly higher than the utility from not reporting:

$$+ \Pr(v = I \mid H, h, \mathit{weak}) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = \mathit{weak})$$
$$+ \Pr(v = I \mid H, l, \mathit{weak}) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = \mathit{weak})$$
$$+ \Pr(v = I \mid H, h, \mathit{strong}) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = \mathit{strong})$$
$$+ \Pr(v = I \mid H, l, \mathit{strong}) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = \mathit{strong})$$
$$\geq \Pr(v = I \mid L, h) \Pr(s_3 = h \mid s_1 = H) + \Pr(v = I \mid L, l) \Pr(s_3 = l \mid s_1 = H)$$

To analyze the voter's choice when no scandal is disclosed, we focus on $\Pr(v = I \mid L, h)$ and $\Pr(v = I \mid L, l)$. There are four possibilities:

(i) $\Pr(v = I \mid L, h) = \Pr(v = I \mid L, l) = 1$. Then both types of the incumbent can guarantee getting elected without reporting the scandal, hence neither has an incentive to report one. This cannot be a separating equilibrium.

(ii) $\Pr(v = I \mid L, h) = \Pr(v = I \mid L, l) = 0$. Then neither type of incumbent ever gets elected when not reporting the scandal, and the separating equilibrium does not exist (can be a mixed strategy).

(iii) $\Pr(v = I \mid L, h) = 1$ and $\Pr(v = I \mid L, l) = 0$. This cannot arise as it would require $u_0 > u(L, h)$ and $u_0 < u(L, l)$, whereas $u(L, h) > u(L, l)$.

(iv) $\Pr(v = I \mid L, h) = 0$ and $\Pr(v = I \mid L, l) = 1$. Then we have $u(L, h) > u_0 > u(L, l)$, which is possible.

Based on case (iv) above, the advantaged type's incentive-compatibility (IC) constraint becomes:

$$\begin{aligned}
&\Pr(s_3 = l \mid s_1 = L)\\
&\geq \Pr(v = I \mid H, h, strong)\Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = strong)\\
&+ \Pr(v = I \mid H, l, strong)\Pr(s_3 = l \mid s_1 = L)\Pr(s_2 = strong)\\
&+ \Pr(v = I \mid H, h, weak)\Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = weak)\\
&+ \Pr(v = I \mid H, l, weak)\Pr(s_3 = l \mid s_1 = L)\Pr(s_2 = weak)
\end{aligned}$$

Also, the disadvantaged type's IC constraint becomes:

$$\begin{aligned}
&\Pr(v = I \mid H, h, strong)^{(1)}\Pr(s_3 = h \mid s_1 = H)\Pr(s_2 = strong)\\
&+ \Pr(v = I \mid H, l, strong)^{(2)}\Pr(s_3 = l \mid s_1 = H)\Pr(s_2 = strong)\\
&+ \Pr(v = I \mid H, h, weak)^{(3)}\Pr(s_3 = h \mid s_1 = H)\Pr(s_2 = weak)\\
&+ \Pr(v = I \mid H, l, weak)^{(4)}\Pr(s_3 = l \mid s_1 = H)\Pr(s_2 = weak)\\
&\geq \Pr(s_3 = l \mid s_1 = H)
\end{aligned}$$

We know (3) = 0, as $u(L, h) > u_0$ implies $u(H, h, weak) > u_0$. We also know that (1) = 1 implies (2) = 1. We can't have only (2) = 1 as then the advantaged type prefers not to report the scandal. If (2) = (4) = 1, then the incumbent is indifferent between sending or not sending the scandal, regardless of the realization of s_1. This is not an interesting case, as it means the scandal doesn't affect the outcome. If (1) = (2) = (4) = 1, then the disadvantaged type prefers to send a scandal. Hence, the separating equilibrium should have (1) = (2) = 1 only: the voter votes for the incumbent whenever the scandal signal turns out to be strong, and votes for the challenger otherwise. We will explain this in detail below.

To analyze the voter's choice when scandal is disclosed, we focus on $\Pr(v = I \mid H, h, strong)$, $\Pr(v = I \mid H, l, strong)$, $\Pr(v = I \mid H, h, weak)$, and $\Pr(v = I \mid H, l, weak)$.

We have $\Pr(v = I \mid H, h, weak) = 0$.

Proof. Since $p_\theta \in (0,1)$ and $\pi_1, \pi_3 \in (0.5,1)$, then $p_\theta^{H,h} > p_\theta^{L,h}$, which implies that $u(H,h) > u(L,h)$. Since $p_\omega \in (0,1)$ and $\pi_2 \in (0.5,1)$, then $p_\omega^w < p_\omega$, which implies that $u(H,h,weak) > u(H,h)$. Thus, we have $u(H,h,weak) > u(L,h)$. The assumption $u(L,h) > u_0$ implies $u(H,h,weak) > u_0$, which leads to $\Pr(v = I \mid H,h,weak) = 0$. □

Now consider $\Pr(v = I \mid H,h,strong)$, $\Pr(v = I \mid H,l,strong)$, and $\Pr(v = I \mid H,l,weak)$.

Given that $\Pr(v = I \mid H,l,weak) = 1$ implies $\Pr(v = I \mid H,l,strong) = 1$, and $\Pr(v = I \mid H,h,strong) = 1$ implies $\Pr(v \mid H,l,strong) = 1$, there are five possibilities:

(i) $\Pr(v = I \mid H,l,weak) = \Pr(v = I \mid H,h,strong) = \Pr(v = I \mid H,l,strong) = 1$. Then, the advantaged type's IC constraint is violated, since

$$\Pr(s_3 = l \mid s_1 = L) < \Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = strong) + \Pr(s_3 = l \mid s_1 = L).$$

(ii) $\Pr(v = I \mid H,h,strong) = 0, \Pr(v = I \mid H,l,weak) = \Pr(v = I \mid H,l,strong) = 1$. This implies that $u_0 < u(H,h,strong)$ and $u_0 > u(H,l,weak) > u(H,l,strong)$. In this separating equilibrium, both types of the incumbent are indifferent between reporting and not reporting the scandal. This is not an interesting case.

(iii) $\Pr(v = I \mid H,h,strong) = 0, \Pr(v = I \mid H,l,strong) = 1, \Pr(v = I \mid H,l,weak) = 0$. Then, the disadvantaged type's IC constraint is violated, since $\Pr(s_3 = l \mid s_1 = H)\Pr(s_2 = strong) < \Pr(s_3 = l \mid s_1 = H)$.

(iv) $\Pr(v = I \mid H,l,strong) = \Pr(v = I \mid H,h,strong) = \Pr(v = I \mid H,l,weak) = 0$. Then, the disadvantaged type's IC constraint is violated, since $0 < \Pr(s_3 = l \mid s_1 = H)$.

(v) $\Pr(v = I \mid H,h,strong) = \Pr(v = I \mid H,l,strong) = 1$ and $\Pr(v = I \mid H,l,weak) = 0$. This implies that $u_0 > u(H,h,strong) > u(H,l,strong)$ and $u_0 < u(H,l,weak)$. The IC constraints are reduced into $\Pr(s_3 = l \mid s_1 =$

$H) \leq \Pr(s_2 = strong) \leq \Pr(s_3 = l \mid s_1 = L)$. This possibility is valid since $\Pr(s_3 = l \mid s_1 = H) \leq \Pr(s_3 = l \mid s_1 = L)$.

No Other Separating Equilibrium

Now consider the separating equilibrium in which the disadvantaged type never reports the scandal, $\Pr(R = 0 \mid s_1 = H) = 1$, while the advantaged type always reports the scandal, $\Pr(R = 1 \mid s_1 = L) = 1$. We will demonstrate that this separating equilibrium cannot exist.

The advantaged type ($s_1 = L$) prefers to report the scandal if the utility from reporting is weakly higher than the utility from not reporting:

$$\begin{aligned}
&\Pr(v = I \mid L, h, strong) \Pr(s_3 = h \mid s_1 = L) \Pr(s_2 = strong) \\
&+ \Pr(v = I \mid L, l, strong) \Pr(s_3 = l \mid s_1 = L) \Pr(s_2 = strong) \\
&+ \Pr(v = I \mid L, h, weak) \Pr(s_3 = h \mid s_1 = L) \Pr(s_2 = weak) \\
&+ \Pr(v = I \mid L, l, weak) \Pr(s_3 = l \mid s_1 = L) \Pr(s_2 = weak) \\
&\geq \Pr(v = I \mid H, h) \Pr(s_3 = h \mid s_1 = L) + \Pr(v = I \mid H, l) \Pr(s_3 = l \mid s_1 = L)
\end{aligned}$$

The disadvantaged type ($s_1 = H$) prefers not to report the scandal if the utility from not reporting is weakly higher than the utility from reporting:

$$\begin{aligned}
&\Pr(v = I \mid H, h) \Pr(s_3 = h \mid s_1 = H) + \Pr(v = I \mid H, l) \Pr(s_3 = l \mid s_1 = H) \\
&\geq \Pr(v = I \mid L, h, strong) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = strong) \\
&+ \Pr(v = I \mid L, l, strong) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = strong) \\
&+ \Pr(v = I \mid L, h, weak) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = weak) \\
&+ \Pr(v = I \mid L, l, weak) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = weak)
\end{aligned}$$

First discuss $\Pr(v = I \mid H, h)$ and $\Pr(v = I \mid H, l)$, and there are four possibilities:

(i) $\Pr(v = I \mid H, h) = \Pr(v = I \mid H, l) = 1$. Then, both types of the incumbent will never report the scandal. This cannot be a separating equilibrium.

(ii) $\Pr(v = I \mid H, h) = \Pr(v = I \mid H, l) = 0$. Then, both types of the incumbent always report the scandal. This cannot be a separating equilibrium.

(iii) $\Pr(v = I \mid H, h) = 1, \Pr(v = I \mid H, l) = 0$. This leads to a contradiction where $u_0 > u(H, h)$ and $u(H, l) > u_0$, since we have $u(H, h) > u(H, l)$.

(iv) $\Pr(v = I \mid H, h) = 0, \Pr(v = I \mid H, l) = 1$. This case is possible if we have $u(H, l) < u_0 < u(H, h)$.

Based on the case (iv) above, the advantaged type's IC constraint becomes:

$$\begin{aligned}
&\Pr(v = I \mid L, h, \mathit{strong}) \Pr(s_3 = h \mid s_1 = L) \Pr(s_2 = \mathit{strong}) \\
&+ \Pr(v = I \mid L, l, \mathit{strong}) \Pr(s_3 = l \mid s_1 = L) \Pr(s_2 = \mathit{strong}) \\
&+ \Pr(v = I \mid L, h, \mathit{weak}) \Pr(s_3 = h \mid s_1 = L) \Pr(s_2 = \mathit{weak}) \\
&+ \Pr(v = I \mid L, l, \mathit{weak}) \Pr(s_3 = l \mid s_1 = L) \Pr(s_2 = \mathit{weak}) \\
&\geq \Pr(s_3 = l \mid s_1 = L)
\end{aligned}$$

The disadvantaged type's IC constraint becomes:

$$\begin{aligned}
&\Pr(s_3 = l \mid s_1 = H) \\
&\geq \Pr(v = I \mid L, h, \mathit{strong}) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = \mathit{strong}) \\
&+ \Pr(v = I \mid L, l, \mathit{strong}) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = \mathit{strong}) \\
&+ \Pr(v = I \mid L, h, \mathit{weak}) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = \mathit{weak}) \\
&+ \Pr(v = I \mid L, l, \mathit{weak}) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = \mathit{weak})
\end{aligned}$$

Now consider $\Pr(v = I \mid L, h, \mathit{strong})$, $\Pr(v = I \mid L, l, \mathit{strong})$, $\Pr(v = I \mid L, h, \mathit{weak})$ and $\Pr(v = I \mid L, l, \mathit{weak})$.

First, we have $\Pr(v = I \mid L, l, \mathit{strong}) = 1$, since $u(H, l) > u(L, l, \mathit{strong})$ and $\Pr(v = I \mid H, l) = 1$.

$\Pr(v = I \mid L, h, \mathit{weak}) = 1$ implies $\Pr(v = I \mid L, l, \mathit{weak}) = 1$ because if $u_0 > u(L, h, \mathit{weak})$, then $u_0 > u(L, l, \mathit{weak})$, since $u(L, h, \mathit{weak}) > u(L, l, \mathit{weak})$.

$\Pr(v = I \mid L, h, weak) = 1$ implies $\Pr(v = I \mid L, h, strong) = 1$, because if $u_0 > u(L, h, weak)$, then $u_0 > u(L, h, strong)$, since $u(L, h, weak) > u(L, h, strong)$.

For $\Pr(v = I \mid L, h, strong)$, $\Pr(v = I \mid L, h, weak)$ and $\Pr(v = I \mid L, l, weak)$, there are five possibilities:

(i) $\Pr(v = I \mid L, h, strong) = \Pr(v = I \mid L, h, weak) = \Pr(v = I \mid L, l, weak) = 1$. Then, both types of the incumbent will always report the scandal, and there is no separating equilibrium.

(ii) $\Pr(v = I \mid L, h, weak) = 0, \Pr(v = I \mid L, h, strong) = 1, \Pr(v = I \mid L, l, weak) = 1$. Then the disadvantaged type's IC constraint is violated, since

$$\Pr(s_3 = l \mid s_1 = H) < \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = strong) + \Pr(s_3 = l \mid s_1 = H).$$

(iii) $\Pr(v = I \mid L, h, weak) = 0, \Pr(v = I \mid L, h, strong) = 1, \Pr(v = I \mid L, l, weak) = 0$. This leads to a contradiction where $\Pr(s_2 = strong) \geq \Pr(s_3 = l \mid s_1 = L)$ and $\Pr(s_3 = l \mid s_1 = H) \geq \Pr(s_2 = strong)$, since we have $\Pr(s_3 = l \mid s_1 = L) > \Pr(s_3 = l \mid s_1 = H)$.

(iv) $\Pr(v = I \mid L, h, weak) = 0, \Pr(v = I \mid L, h, strong) = 0, \Pr(v = I \mid L, l, weak) = 1$. This separating equilibrium is trivial, since both types of the incumbent are indifferent between reporting and not reporting the scandal.

(v) $\Pr(v = I \mid L, h, weak) = \Pr(v = I \mid L, h, strong) = \Pr(v = I \mid L, l, weak) = 0$. Then, the advantaged type's IC constraint is violated, since

$$\Pr(s_3 = l \mid s_1 = L) \Pr(s_2 = strong) < \Pr(s_3 = l \mid s_1 = L).$$

To conclude, there is a unique separating equilibrium in which $\Pr(R = 0 \mid s_1 = L) = \Pr(R = 1 \mid s_1 = H) = 1$.

A.2.2 Pooling Equilibria on Revealing

We now discuss pooling equilibria in which both types of the incumbent report the scandal: $\Pr(R = 1 \mid s_1 = L) = \Pr(R = 1 \mid s_1 = H) = 1$.

Let $\Pr(v = I \mid s_2, s_3)$ be the probability that the incumbent is elected ($v = I$) on the equilibrium path ($R = 1$), after the voter observes the signal $s_3 \in \{h, l\}$ about the challenger's ability.

$\Pr(v = I \mid s_2, s_3) = 1(0)$ if and only if the voter's utility u_0 from the incumbent, is higher (lower) than the utility, $u(s_2, s_3)$, from the challenger, where $u(s_2, s_3) = \Pr(s_1 = H \mid s_3)u(H, s_2, s_3) + \Pr(s_1 = L \mid s_3)u(L, s_2, s_3)$.

Let q be the voter's off-equilibrium belief of $s_1 = H$ when the incumbent deviates by not reporting: $q \triangleq \Pr(s_1 = H \mid R = 0)$.

Let $\Pr(v = I \mid q, s_3)$ be the probability that the incumbent is elected ($v = I$) off-equilibrium path ($R = 0$), after the voter observes the signal $s_3 \in \{h, l\}$ about the challenger's ability.

$\Pr(v = I \mid q, s_3) = 1(0)$ if and only if the voter's utility, u_0, from the incumbent is higher (lower) than the utility, $u(s_3)$, from the challenger, where $u(q, s_3) = qu(H, s_3) + (1 - q)u(L, s_3)$.

The disadvantaged type prefers to report the scandal if the equilibrium utility is weakly higher than that from deviation:

$$
\begin{aligned}
&\Pr(v = I \mid h, strong) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = strong) \\
&+ \Pr(v = I \mid l, strong) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = strong) \\
&+ \Pr(v = I \mid h, weak) \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = weak) \\
&+ \Pr(v = I \mid l, weak) \Pr(s_3 = l \mid s_1 = H) \Pr(s_2 = weak) \\
&\geq \Pr(v = I \mid q, h) \Pr(s_3 = h \mid s_1 = H) + \Pr(v = I \mid q, l) \Pr(s_3 = l \mid s_1 = H)
\end{aligned}
$$

The advantaged type prefers to report the scandal if the equilibrium utility is weakly higher than that from deviation:

$$\begin{aligned}
&\Pr(v = I \mid h, strong)\Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = strong)\\
&+ \Pr(v = I \mid l, strong)\Pr(s_3 = l \mid s_1 = L)\Pr(s_2 = strong)\\
&+ \Pr(v = I \mid h, weak)\Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = weak)\\
&+ \Pr(v = I \mid l, weak)\Pr(s_3 = l \mid s_1 = L)\Pr(s_2 = weak)\\
&\geq \Pr(v = I \mid q, h)\Pr(s_3 = h \mid s_1 = L) + \Pr(v = I \mid q, l)\Pr(s_3 = l \mid s_1 = L)
\end{aligned}$$

We consider the pooling equilibria that can coexist with the separating equilibrium.

To analyze the voter's decision when the incumbent deviates by not reporting the scandal, we focus on $\Pr(v = I \mid q, h)$, and $\Pr(v = I \mid q, l)$, and there are four possibilities:

(i) $\Pr(v = I \mid q, h) = \Pr(v = I \mid q, l) = 1$. Then both types of the incumbent will be elected upon deviation to not reporting, so pooling on revealing does not exist.

(ii) $\Pr(v = I \mid q, h) = \Pr(v = I \mid q, l) = 0$. Under the assumption $u(H, l) < u_0$, we have $u(q, l) < u_0$, which implies that upon deviation, for any belief, the voter should elect the incumbent whenever $s_3 = l$. This possibility cannot exist.

(iii) $\Pr(v = I \mid q, h) = 1, \Pr(v = I \mid q, l) = 0$. This cannot arise as it leads to a contradiction where $u(q, h) < u_0$ and $u(q, l) > u_0$.

(iv) $\Pr(v = I \mid q, h) = 0, \Pr(v = I \mid q, l) = 1$. Under the assumption $u(H, l) < u_0$, we have $u(q, l) < u_0$, which implies that upon deviation, for any belief, the incumbent is elected whenever $s_3 = l$. Under the assumption $u(L, h) > u_0$, we have $u(q, h) > u_0$, which implies that upon deviation, for any belief, the challenger is elected whenever $s_3 = h$. This possibility can exist.

To analyze the voter's equilibrium actions, we focus $\Pr(v = I \mid h, strong)$, $\Pr(v = I \mid h, weak)$, $\Pr(v = I \mid l, strong)$, and $\Pr(v = I \mid l, weak)$.

When the separating equilibrium exists, $\Pr(v = I \mid h, strong) = \Pr(v = I \mid l, strong) = 1$ and $\Pr(v = I \mid h, weak) = 0$.

Proof. When the separating equilibrium exists, we have $u(H, h, strong) < u_0$, which implies $u(h, strong) < u_0$ and thus $\Pr(v = I \mid h, strong) = 1$. This in return implies $\Pr(v = I \mid l, strong) = 1$, since $u(l, strong) < u(h, strong)$. Also, $u(L, h) > u_0$ implies $u(h, weak) > u_0$, since $u(h, weak) > u(L, h)$. Thus, we have $\Pr(v = I \mid h, weak) = 0$. □

So the only variable that isn't determined is $\Pr(v = I \mid l, weak)$, and there are two possibilities:

(i) $\Pr(v = I \mid l, weak) = 1$. This implies $u(l, weak) < u_0$. The disadvantaged type's IC constraint always holds, since

$$\Pr(s_3 = h \mid s_1 = H)\Pr(s_2 = strong) + \Pr(s_3 = l \mid s_1 = H) > \Pr(s_3 = l \mid s_1 = H).$$

Also, the advantaged type's IC constraint always holds, since:

$$\Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = strong) + \Pr(s_3 = l \mid s_1 = L) > \Pr(s_3 = l \mid s_1 = L).$$

This possibility may coexist with the separating equilibrium.

(ii) $\Pr(v = I \mid l, weak) = 0$. This implies $u(l, weak) > u_0$. The equilibrium payoff is the same for both types of the incumbent: $\Pr(s_2 = strong)$. The advantaged type ($s_1 = L$) receives a weakly higher utility from deviating to not reporting than the disadvantaged one ($s_1 = H$) does, since $\Pr(s_3 = l \mid s_1 = L) > \Pr(s_3 = l \mid s_1 = H)$. Thus, the advantaged type has more incentive to deviate. The IC constraint is then $\Pr(s_2 = strong) > \Pr(s_3 = l \mid s_1 = L)$, which is the region for which this type of pooling equilibrium exists.

To conclude, we summarize the pooling on revealing equilibria:

The incumbent is elected whenever $s_2 = strong$ regardless of s_3, and the challenger is elected whenever $s_3 = h, s_2 = weak$:

$\Pr(v = I \mid h, strong) = 1$, $\Pr(v = I \mid l, strong) = 1$, and $\Pr(v = I \mid h, weak) = 0$.

Upon deviation, the incumbent is elected whenever $s_3 = l$, while the challenger is elected whenever $s_3 = h$:

$\Pr(v = I \mid l) = 1$ and $\Pr(v = I \mid h) = 0$.

- $\Pr(v = I \mid l, weak) = 1$ in which case $u(l, weak) < u_0$ and the pooling equilibrium always exists and can coexist with the separating equilibrium.
- $\Pr(v = I \mid l, weak) = 0$, in which case $u(l, weak) > u_0$ and the pooling equilibrium exists if $\Pr(s_2 = strong) \geq \Pr(s_3 = L \mid s_1 = l)$ but cannot coexist with the separating equilibrium.

A.2.3 Pooling Equilibria on Not Revealing

We now discuss pooling equilibria in which neither type of the incumbent reports a scandal: $\Pr(R = 0 \mid s_1 = L) = \Pr(R = 0 \mid s_1 = H) = 1$.

Let $\Pr(v = I \mid s_3)$ be the probability that the incumbent is elected ($v = I$) on the equilibrium path ($R = 0$), after the voter observes the signal $s_3 \in \{h, l\}$ about the challenger's ability.

$\Pr(v = I \mid s_3) = 1(0)$ if and only if the voter's utility, u_0, from the incumbent is higher (lower) than the utility, $u(s_3)$, from the challenger, where $u(s_3) = \Pr(s_1 = H \mid s_3)u(H, s_3) + \Pr(s_1 = L \mid s_3)u(L, s_3)$.

Let q be the voter's off-equilibrium belief of $s_1 = H$ when the incumbent deviates reporting: $q \triangleq \Pr(s_1 = H \mid R = 1)$.

Let $\Pr(v = I \mid q, s_2, s_3)$ be the probability that the incumbent is elected ($v = I$) off-equilibrium ($R = 1$), after the voter observes the signal $s_3 \in \{h, l\}$ about the challenger's ability, and the signal $s_2 \in \{strong, weak\}$ about the challenger's corruption.

$\Pr(v = I \mid q, s_2, s_3) = 1(0)$ if and only if the voter's utility u_0 from the incumbent, is higher (lower) than the utility, $u(s_2, s_3)$, from the challenger, where $u(s_2, s_3) = qu(H, s_2, s_3) + (1-q)u(L, s_2, s_3)$.

The disadvantaged type prefers not to report if the equilibrium utility is weakly higher than that from deviation:

$$
\begin{aligned}
&\Pr(v = I \mid h)\Pr(s_3 = h \mid s_1 = H) + \Pr(v = I \mid l)\Pr(s_3 = l \mid s_1 = H) \\
&\geq \Pr(v = I \mid q, h, strong)\Pr(s_3 = h \mid s_1 = H)\Pr(s_2 = strong) \\
&+ \Pr(v = I \mid q, l, strong)\Pr(s_3 = l \mid s_1 = H)\Pr(s_2 = strong) \\
&+ \Pr(v = I \mid q, h, weak)\Pr(s_3 = h \mid s_1 = H)\Pr(s_2 = weak) \\
&+ \Pr(v = I \mid q, l, weak)\Pr(s_3 = l \mid s_1 = H)\Pr(s_2 = weak)
\end{aligned}
$$

The advantaged type prefers not to report if the equilibrium utility is weakly higher than that from deviation:

$$
\begin{aligned}
&\Pr(v = I \mid h)\Pr(s_3 = h \mid s_1 = L) + \Pr(v = I \mid l)\Pr(s_3 = l \mid s_1 = L) \\
&\geq \Pr(v = I \mid q, h, strong)\Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = strong) \\
&+ \Pr(v = I \mid q, l, strong)\Pr(s_3 = l \mid s_1 = L)\Pr(s_2 = strong) \\
&+ \Pr(v = I \mid q, h, weak)\Pr(s_3 = h \mid s_1 = L)\Pr(s_2 = weak) \\
&+ \Pr(v = I \mid q, l, weak)\Pr(s_3 = l \mid s_1 = L)\Pr(s_2 = weak)
\end{aligned}
$$

To analyze the voter's decision on the equilibrium path, we focus on $\Pr(v = I \mid h)$ and $\Pr(v = I \mid l)$, and there are four possibilities.

(i) $\Pr(v = I \mid h) = \Pr(v = I \mid l) = 1$. This implies that $u(h) < u_0$, $u(l) < u_0$: both types of the incumbent will be elected if no scandal is reported. So no one deviates to reporting (unless deviation always leads the voter to vote incumbent and there is indifference), and pooling on not revealing always exists for any out-of-equilibrium belief of the voter upon observing a deviation to reporting a scandal. This pooling equilibrium cannot coexist with the separating equilibrium in which we have $u(L,h) > u_0$.

(ii) $\Pr(v = I \mid h) = \Pr(v = I \mid l) = 0$. This implies that $u(h) > u_0$, $u(l) > u_0$: neither type of incumbents will be elected in equilibrium when no scandal is reported. So either type could have an incentive to deviate to reporting if that makes the voter vote for the incumbent at least for some signal realizations, which is always when the separating equilibrium exists under condition $u(H,h,strong) < u_0$ because, for any q, we have $u(q,h,strong) < u_0$.

(iii) $\Pr(v = I \mid h) = 1, \Pr(v = I \mid l) = 0$. This leads to a contradiction where $u(h) < u_0$ and $u(l) > u_0$, because we have $u(h) > u(l)$.

(iv) $\Pr(v = I \mid h) = 0, \Pr(v = I \mid l) = 1$. This implies that $u(h) > u_0 > u(l)$. This may coexist with the separating equilibrium.

To analyze the voter's off-equilibrium actions, we focus on $\Pr(v = I \mid q,h,strong)$, $\Pr(v = I \mid q,h,weak)$, $\Pr(v = I \mid q,l,strong)$, and $\Pr(v = I \mid q,l,weak)$.

We have $\Pr(v = I \mid q,h,strong) = \Pr(v = I \mid q,l,strong) = 1$, when the separating equilibrium exists.

Proof. When the separating equilibrium exists, we have $u(H,h,strong) < u_0$. Then, for any $q \in (0,1)$, we have $u(q,h,strong) < u_0$ and $u(q,l,strong) < u_0$. Hence, upon deviation to reporting, $\Pr(v = I \mid q,h,strong) = \Pr(v = I \mid q,l,strong) = 1$ for any possible belief q. □

$\Pr(v = I \mid q,h,weak) = 1$ implies $\Pr(v = I \mid q,l,weak) = 1$. For $\Pr(v = I \mid q,h,weak)$ and $\Pr(v = I \mid q,l,weak)$, there are three possibilities:

(i) $\Pr(v = I \mid q,h,weak) = \Pr(v = I \mid q,l,weak) = 0$. This implies $u(q,l,weak) > u_0$, and the disadvantaged type's IC constraint becomes $\Pr(s_3 = l \mid s_1 = H) >$

$\Pr(s_2 = \mathit{strong})$. This possibility cannot coexist with the separating equilibrium.

(ii) $\Pr(v = I \mid q, h, \mathit{weak}) = 0$, $\Pr(v = I \mid q, l, \mathit{weak}) = 1$. Then, the disadvantaged type's IC constraint is violated, since $\Pr(s_3 = l \mid s_1 = H) > \Pr(s_3 = l \mid s_1 = H) + \Pr(s_3 = h \mid s_1 = H) \Pr(s_2 = \mathit{strong})$. This possibility cannot exist.

(iii) $\Pr(v = I \mid q, h, \mathit{weak}) = \Pr(v = I \mid q, l, \mathit{weak}) = 1$. This implies that upon deviation, the incumbent is elected, which violates both types' IC constraints. Thus, this possibility cannot exist.

The pooling equilibrium on not revealing cannot coexist with the separating equilibrium.

A.3 Comparative Statics

Threshold Values

$\frac{\partial \underline{\lambda}}{\partial p_\theta} = \frac{\pi_1(\pi_1-1)(2\pi_3-1)}{(p_\theta+\pi_1-2p_\theta\pi_1-1)^2} < 0$, since $\pi_1 \in (0.5, 1)$ and $\pi_3 \in (0.5, 1)$.

$\frac{\partial \underline{\lambda}}{\partial \pi_1} = \frac{p_\theta(p_\theta-1)(2\pi_3-1)}{(p_\theta+\pi_1-2p_\theta\pi_1-1)^2} < 0$, since $p_\theta \in (0, 1)$ and $\pi_3 \in (0.5, 1)$.

$\frac{\partial \underline{\lambda}}{\partial \pi_3} = \frac{1-\pi_1-p_\theta}{p_\theta(2\pi_1-1)+1-\pi_1} < 0 (> 0)$, if $p_\theta + \pi_1 > 1 (< 1)$.

$\frac{\partial^2 \underline{\lambda}}{\partial \pi_1 \partial \pi_3} = \frac{2p_\theta(p_\theta-1)}{(p_\theta+\pi_1-2p_\theta\pi_1-1)^2} < 0$, since $p_\theta \in (0, 1)$.

$\frac{\partial \overline{\lambda}}{\partial p_\theta} = \frac{-\pi_1(1-\pi_1)(2\pi_3-1)}{(p_\theta+\pi_1-2p_\theta\pi_1)^2} < 0$, since $\pi_1 \in (0.5, 1)$ and $\pi_3 \in (0.5, 1)$.

$\frac{\partial \overline{\lambda}}{\partial \pi_1} = \frac{p_\theta(1-p_\theta)(2\pi_3-1)}{(p_\theta+\pi_1-2p_\theta\pi_1)^2} > 0$, since $p_\theta \in (0, 1)$ and $\pi_3 \in (0.5, 1)$.

$\frac{\partial \overline{\lambda}}{\partial \pi_3} = \frac{\pi_1-p_\theta}{p_\theta+\pi_1-2p_\theta\pi_1} > 0 (< 0)$, if $\pi_1 > p_\theta$(if $\pi_1 < p_\theta$), where the denominator $p_\theta + \pi_1 - 2p_\theta\pi_1$ can be written as $(p_\theta - \pi_1)^2 + p_\theta - {p_\theta}^2 + \pi_1 - {\pi_1}^2 > 0$. This means reporting a scandal is more likely for higher π_3 when p_θ is high, while it is less likely for higher π_3 when p_θ is low.

$\frac{\partial^2 \overline{\lambda}}{\partial \pi_1 \partial \pi_3} = \frac{2p_\theta(1-p_\theta)}{(p_\theta+\pi_1-2p_\theta\pi_1)^2} > 0$, since $p_\theta \in (0, 1)$

We have

$$\frac{\partial D}{\partial p_\theta} = \frac{(2p_\theta - 1)\pi_1(\pi_1 - 1)(2\pi_1 - 1)(2\pi_3 - 1)}{(p_\theta + \pi_1 - 2p_\theta\pi_1)^2(1 - p_\theta - \pi_1 + 2p_\theta\pi_1)^2}.$$

Then, $\frac{\partial D}{\partial p_\theta} > 0$ if $p_\theta < 0.5$ and $\frac{\partial D}{\partial p_\theta} < 0$ if $p_\theta > 0.5$.

We have

$$\frac{\partial D}{\partial \pi_1} = \frac{(1 - p_\theta)p_\theta(1 - 2p_\theta(1 - 2\pi_1)^2 + 2{p_\theta}^2(1 - 2\pi_1)^2 - 2(1 - \pi_1)\pi_1)(2\pi_3 - 1)}{(p_\theta + \pi_1 - 2p_\theta\pi_1)^2(1 - p_\theta - \pi_1 + 2p_\theta\pi_1)^2}.$$

Consider the expression in the numerator, where:

$$1 - 2p_\theta(1 - 2\pi_1)^2 + 2{p_\theta}^2(1 - 2\pi_1)^2 - 2(1 - \pi_1)\pi_1.$$

Since

$$-2p_\theta(1 - 2\pi_1)^2 + 2{p_\theta}^2(1 - 2\pi_1)^2 = -2(1 - p_\theta)p_\theta(1 - 2\pi_1)^2,$$

this expression can be simplified into

$$1 - 2(1 - p_\theta)p_\theta(1 - 2\pi_1)^2 - 2(1 - \pi_1)\pi_1.$$

Since

$$1 - 2(1 - \pi_1)\pi_1 = (1 - 2\pi_1)^2 + 2\pi_1 - 2{\pi_1}^2,$$

then the expression becomes

$$(1 - 2(1 - p_\theta)p_\theta)(1 - 2\pi_1)^2 + 2\pi_1 - 2{\pi_1}^2,$$

which can be simplified into

$$((1 - p_\theta)^2 + {p_\theta}^2)(1 - 2\pi_1)^2 + 2\pi_1 - 2{\pi_1}^2 > 0,$$

which makes the numerator positive. Given the denominator is positive, the whole expression is positive. So the separating region widens as π_1 increases.

Also, we have $\frac{\partial D}{\partial \pi_3} > 0$. Note that under the assumption $p_\theta = 0.5$, we will have

$\frac{\partial \underline{\lambda}}{\partial \pi_3} < 0$ and $\frac{\partial \bar{\lambda}}{\partial \pi_3} > 0$.

Comparing the voter's payoff under the separating equilibrium and pooling on revealing

The voter's payoff in the separating equilibrium is given by:

$$(\Pr(s_1 = H)\Pr(s_2 = strong) + \Pr(s_1 = L)\Pr(s_3 = l \mid s_1 = L))u_0 +$$
$$\Pr(s_1 = H)\Pr(s_2 = weak)(k{p_\theta}^H - {p_\omega}^w) + \Pr(s_1 = L)\Pr(s_3 = h \mid s_1 = L)(k{p_\theta}^{Lh} - p_\omega).$$

As π_1 increases, $\Pr(s_3 = l \mid s_1 = L)$ also increases, making the separating equilibrium more likely. At the critical point where $\Pr(s_2 = strong) = \Pr(s_3 = l \mid s_1 = L)$, we can rewrite the voter's payoff in the separating equilibrium as:

$$\Pr(s_2 = strong)u_0 + \Pr(s_1 = H)\Pr(s_2 = weak)(k{p_\theta}^H - {p_\omega}^w) +$$
$$\Pr(s_1 = L)\Pr(s_2 = weak)(k{p_\theta}^{Lh} - p_\omega).$$

The voter receives a lower payoff in the separating equilibrium than in the pooling equilibrium on revealing if and only if:

$$\Pr(s_1 = H)(k{p_\theta}^H - {p_\omega}^w) + \Pr(s_1 = L)(k{p_\theta}^{Lh} - p_\omega) < kp_\theta - {p_\omega}^w.$$

Rearranging gives:

$$\frac{\Pr(s_1 = H)}{\Pr(s_1 = L)}(k{p_\theta}^H - {p_\omega}^w) + (k{p_\theta}^{Lh} - p_\omega) < (kp_\theta - {p_\omega}^w) + \frac{\Pr(s_1 = H)}{\Pr(s_1 = L)}(kp_\theta - {p_\omega}^w).$$

This simplifies to:

$$k < \frac{p_\omega - {p_\omega}^w}{p_\theta^{Lh} - p_\theta + (p_\theta^H - p_\theta)\frac{\Pr(s_1=H)}{\Pr(s_1=L)}}.$$

When $p_\theta = p_\omega = 0.5$, the condition becomes

$$k < \frac{\pi_2 - 0.5}{\pi_3 - 0.5},$$

indicating that the voter's payoff in pooling on revealing is higher than in separating as π_1 increases. In this case, $\Pr(s_2 = strong) = \Pr(l \mid L)$ implies that $\pi_1 + \pi_3 - 2\pi_1\pi_3 = 0.5$.

Note: if we assume $p_\theta = 0.5$, the analysis for π_3 mirrors that of π_1, as $\Pr(l \mid L)$ increases with π_3 if and only if $\pi_1 > p_\theta$. An increase in π_3 has two effects on the voter: 1) the voter's own signal about ability becomes more informative, and 2) the equilibrium strategy reveals the incumbent's private signal about ability.

Whenever $\pi_2 > \pi_3$, this holds true if $k \leq 1$. This implies that if the precision of scandal is sufficiently high, and k is not too high, then pooling on revealing may dominate the separating equilibrium. This is natural as the information provided by the scandal becomes more important.

Hence, having more information (higher π_3) may indeed hurt the voter if k is low enough (indicating that corruption is sufficiently important compared to ability) and the equilibrium moves from pooling on revealing to separating.

The conditions on k are not incompatible with the separating equilibrium.

Comparing the voter's payoff in separating equilibrium and pooling on not revealing

The voter's payoff in the pooling equilibrium on not revealing is given by:

$$\Pr(l)u_0 + \Pr(h)(kp_\theta^h - p_\omega).$$

The voter's payoff in the separating equilibrium is given by:

$$\begin{aligned}&[\Pr(s_1 = H)\Pr(s_2 = strong) + \Pr(s_1 = L)\Pr(l \mid L)]u_0 + \\ &\quad \Pr(s_1 = H)\Pr(s_2 = weak)(kp_\theta{}^H - p_\omega{}^w) + \Pr(s_1 = L)\Pr(h \mid L)(kp_\theta{}^{Lh} - p_\omega).\end{aligned}$$

As π_1 increases, $\Pr(l \mid H)$ decreases, making the separating equilibrium more likely. At the critical point where $\Pr(s_2 = strong) = \Pr(l \mid H)$, we can rewrite the voter's payoff in the separating equilibrium as

$$\Pr(l)u_0 + \Pr(s_1 = H)\Pr(h \mid H)(kp_\theta{}^H - p_\omega{}^w) + \Pr(s_1 = L)\Pr(h \mid L)(kp_\theta{}^{Lh} - p_\omega).$$

The voter gets a higher payoff in the separating equilibrium than in the pooling equilibrium on not revealing if and only if:

$$\Pr(H,h)(kp_\theta{}^H - p_\omega{}^w) + \Pr(L,h)(kp_\theta{}^{Lh} - p_\omega) > \Pr(h)(kp_\theta^h - p_\omega),$$

which can be rewritten as:

$$(kp_\theta{}^H - p_\omega{}^w) + \frac{\Pr(L,h)}{\Pr(H,h)}(kp_\theta{}^{Lh} - p_\omega) > (kp_\theta^h - p_\omega) + \frac{\Pr(L,h)}{\Pr(H,h)}(kp_\theta^h - p_\omega),$$

which can be reduced to:

$$k < \frac{p_\omega - p_\omega{}^w}{p_\theta^h - p_\theta^H + (p_\theta^h - p_\theta^{Lh})\frac{\Pr(L,h)}{\Pr(H,h)}}.$$

When $p_\theta = p_\omega = 0.5$, this condition becomes $k < \frac{\pi_2 - 0.5}{2\pi_3 - 1}$, indicating that the voter's payoff in that separating equilibrium is higher than in pooling on not revealing.

Note: if we assume $p_\theta = 0.5$, the analysis for π_3 is the same as above. $\Pr(l \mid H)$ decreases with π_3 if and only if $\pi_1 + p_\theta > 1$

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